$V_{x+k-1} = b_k v q_{x+k-1} + k V_x v p_{x+k-1} - NP_{k-1}$
Table of Contents

Section 1: Reserves ................................................................. 3
  1.1 Reserve Overview .......................................................... 3
  1.2 Calculate Net Premiums .................................................. 3
    1.2.1 Net Level premium calculation ...................................... 3
    1.2.2 Modified net premium calculation ................................. 3
      1.2.2.1 Tabular COI or Basic One Half Cx ................................ 3
      1.2.2.2 Net Level ............................................................... 3
      1.2.2.3 All other modified premium methods ............................ 4
        1.2.2.3.1 19 Pay renewal net premium ................................. 4
        1.2.2.3.2 Death Benefit used in expense allowance ................. 4
        1.2.2.3.3 Illinois Method .................................................. 6
        1.2.2.3.4 Grading and the New Jersey Method ......................... 6
      1.2.3 First year Modified net premium calculation (α) ............. 6
    1.3 Calculate Terminal Reserves .......................................... 6
    1.4 Deficiency Reserves Calculation ..................................... 7
Section 2: Gross Premiums ................................................... 7
Section 3: Commutation Functions ........................................ 8
Section 4: Benefits .............................................................. 8
  4.1 Death Benefit ................................................................. 8
  4.2 Endowment Amount ...................................................... 9
Section 5: Cash Values .......................................................... 9
  5.1 Adjusted premium using the 1980 or the Actuarial Guideline CCC Method ........................................................................ 10
  5.2 Adjusted premium using the 1941 Method ............................ 11
Section 6: Mid Year and Comparisons .................................... 11
  6.1 Interpolation ................................................................ 11
  6.2 MidYear Reserve ........................................................... 12
  6.3 XXX Comparisons .......................................................... 12
    6.3.1 Mid Year, Basic Reserve ............................................... 12
    6.3.2 Mid Year, Minimum Reserve .......................................... 12
    6.3.3 Terminals, Basic and Minimum Reserve ......................... 12
    6.3.4 Net Premium, Basic and Minimum assumptions ............... 12
    6.3.5 Deficiency Reserves ................................................... 12
Section 7: Segmentation (Reserves) ........................................ 12
  7.1 Gross Premium ratios ...................................................... 12
  7.2 Mortality Ratios ............................................................. 13
  7.3 Segmentation test .......................................................... 13
Section 8: Mortality ............................................................. 13
  8.1 General formula ............................................................ 13
  8.2 Last Survivor two lives .................................................. 13
Section 9: Unusual Cash Value Test ....................................... 14
  9.1 Expense Allowance ....................................................... 14
  9.2 Terminal Reserve .......................................................... 14
  9.3 Mid Year Reserve .......................................................... 14
  9.4 Unusual Cash Value Test ................................................ 14
Section 10: Smoothness Test .................................................. 15
  10.1 Nonforfeiture Factors (NFF) ............................................ 15
  10.2 Basic Cash Value Calculation ......................................... 15
  10.3 Segments ................................................................. 15
  10.4 Smoothness Test Determination ...................................... 15
Section 11: Selected References ............................................ 15
Index ................................................................................. 16
Section 1: Reserves

1.1 Reserve Overview

The net premium calculation and terminal reserve functions are always called together with the net premium function being called first. These functions are called each time there is a new segment or if there are new mortality assumptions. Thus, if a XXX methodology is used, the net premium and terminal reserve function will be called for the ½ Cx, basic and minimum assumptions as well as the unitary and segmentation assumptions.

If there is more than one segment in the segmentation calculation, it will be called once for each segment. In addition to identifying the beginning and ending segment, identifying whether an expense allowance is permitted is another input. The expense allowance is only calculated for the first segment.

1.2 Calculate Net Premiums

The net level annual premiums are always calculated regardless of the reserve method chosen. For the modified net premiums, the first year percentage could be different than the renewal percentage because of the expense allowance. The renewal net premium is not allowed to be greater than the first year net premium percentage.

\( r_n \) is the percentage used for the net level calculation (no expense allowance) and \( r_c \) is the percentage for the modified premium assumption.

1.2.1 Net Level premium calculation

\[
\text{If } PVFGP_{\text{Begin Age}} \neq 0, \text{ Then} \\
\quad r_n = \frac{PVFB_{\text{Begin Age}}}{PVFGP_{\text{Begin Age}}} \\
\text{Else} \\
\quad r_n = 0
\]

For all \( x \) less than the premium cease age,

\[
\text{Net Annual Premiums}_x = r_n \times \text{Final Gross Premiums}_{(x - \text{Issue Age})}
\]

\[
\text{Net Annual Premiums}_x = 0 \text{ for all } x \geq \text{ the premium cease age.}
\]

1.2.2 Modified net premium calculation

An expense allowance is allowed only in the first segment. The modified net premium is dependent upon the method chosen.

1.2.2.1 Tabular COI or Basic One Half \( C_x \)

For all \( x \),

\[
\text{If Timing = Continuous Or Timing = Discount Continuous, then} \\
\quad PV \text{ Endowment Benefit} = \text{Endowment Amount}_x \times \frac{D(x+1)}{D_x} \\
\text{Else} \\
\quad PV \text{ Endowment Benefit} = \text{Endowment Amount}_x \times \frac{D(x+1)}{D_x}
\]

\[
\text{If Timing = Curtate, Then} \\
\quad \text{Modified Net Premium}_x = \text{Death Benefit}_x \times \frac{C_x}{D_x} \\
\text{Else If Timing = Semi Continuous, Then} \\
\quad \text{Modified Net Premium}_x = \text{Death Benefit}_x \times \frac{\bar{C}_x}{\bar{D}_x} \\
\text{Else} \\
\quad \text{Modified Net Premium}_x = \text{Death Benefit}_x \times \frac{\bar{C}_x}{\bar{D}_x}
\]

\[
\text{Modified Net Premium}_x = \text{Modified Net Premium}_x + PV \text{ Endowment Benefit}
\]

1.2.2.2 Net Level

For all \( x \),
Modified Net Premium, = Net Annual Premiums

1.2.2.3 All other modified premium methods

All the other modified premium methods require an expense allowance calculation in the first year. Tests must be performed to determine the maximum amount of expense allowance allowed. Also, for non level premiums and/or death benefits, assumptions must be made to the generalized level premium, level death benefit formulae.

1.2.2.3.1 19 Pay renewal net premium

If Timing = Curtate Or Expense Allowance Curtate, Then

\[
\text{Renewal 19 Pay Premium, } = \frac{M_{x+1}}{N_{x+1} - N_{x+20}}
\]

Else If Timing = Semi Continuous, Then

\[
\text{Renewal 19 Pay Premium, } = \frac{\widetilde{M}_{x+1}}{N_{x+1} - N_{x+20}}
\]

Else

\[
\text{Renewal 19 Pay Premium, } = \frac{\widetilde{M}_{x+1}}{N_{x+1} - N_{x+20}}
\]

1.2.2.3.2 Death Benefit used in expense allowance

In order to calculate the expense allowance, a constant death benefit is needed. If the Death Benefit Type is constant, then that death benefit is use. If it is not, then the following calculation is performed:

if 19 pay death benefit definition = Ave DB, then

\[
DB = \frac{\sum_{j=2}^{\min(10, \max \text{dur})} DB_j}{\min(9, \max \text{dur} - 1)}
\]

Else If 19 pay death benefit definition = ELRA, then

\[
DB = \frac{\sum_{j=2}^{x+2} b_{j+2}v^{j+1}q_{x+j} + PVFB(\text{no endowment})_{x+1}}{A} = \frac{\left(M_{x+1} - M_{x+a}\right)}{D_{x+1}}
\]

The ELRA method was proposed by Walter O. Menge in his paper “Commissioners Reserve Valuation Method”, RAIA XXXV (page 283) and discussed in Bowes, Gerber, Hickman, Jones, and Nesbitt "Actuarial Mathematics" Textbook as well as Actuarial Guideline XVII “Calculation of CRVM Reserves When Death Benefits Are Not Level”.

Both methods are consistent with Actuarial Guideline VII (Interpretation Regarding Calculation of Equivalent Level Amounts) in that the endowment benefit, if any, is NOT included in the calculation.

1.2.2.3.3 High premium vs. low premium and \( \beta^{EA} \) determination

For all renewal years, the net premium is a constant % of the gross premium.

\[
NP, = r_c * GP,
\]
Where \( r_c = \frac{PVFB_x}{PVFGP_x} + \text{Expense Allowance} \)

**Expense Allowance timing**

If the user chooses a curtate expense allowance, then all factors in the expense allowance will be curtate. If the user does not choose the curtate expense allowance, then the expense allowance factors will be calculated consistent with the timing chosen. This is consistent with Actuarial Guideline XVIII. Thus, if the timing is semi-continuous and the curtate expense allowance is not checked, then the timing of the expense allowance will be semi-continuous (curtate premiums and immediate payment of claims).

In order to calculate the expense allowance, a test must be performed to determine whether a plan is a low or a high premium plan. The test can be performed by assuming the Menge interpretation or the traditional interpretation.

**Assuming the Menge interpretation.**

Expense Allowance = \( (\beta \cdot DB - \text{One Year Term Cost}) / PVFGP_{\text{Begin Age}} \)

For the DB definition, see section Death Benefit used in expense allowance.

One Year Term Cost is the premium for a one year term insurance accounting for the timing of the premium and benefits in the first year.

\( \beta \) definition depends upon whether the policy is a low or high premium case.

Need to determine whether Full Preliminary Term (low premium) is allowed. If not, the policy is classified as a “High premium” case.

FPT is allowed if \( r_f \cdot G_0 \leq DB \cdot 19P_{x+1} \)

Where \( r_f = \frac{PVFB_{x+1}}{PVFGP_{x+1}} \)

\( 19P_{x+1} = \frac{M_{x+1}}{N_{x+1} + N_{x+20}} \)

The \( M \) commutation function is curtate if the policy assumes curtate assumptions OR if the expense allowance is curtate. Otherwise, \( M \) is a continuous commutation function. \( N \) is a curtate commutation function if \( M \) is curtate or if the policy assumes semi continuous timing assumptions. Otherwise, \( N \) is a continuous commutation function.

For the DB definition, see section Death Benefit used in expense allowance.

For the low premium case (full preliminary term),

\( \beta = r_f \cdot G_0 / DB \)

For the high premium case (full preliminary term is NOT allowed),

\( \beta = 19 \text{ Pay renewal net premium} \) (see section 19 Pay renewal net premium)

For more details, see Actuarial Mathematics by Bowers, Gerber, Hickman, Jones, Nesbitt, 2nd Edition Section 16.9.1

If the traditional interpretation is used,

FPT is allowed if \( \beta \leq DB \cdot 19P_{x+1} \)

Where \( \beta = \frac{PVFB_{(x+1)}}{\bar{a}_{(x+1)}} \)

DB and \( 19P_{x+1} \) are defined similar to the Menge method.
For the low premium case,

$$\text{Expense Allowance} = \frac{\text{PVFB}_{x+1} \div \text{PVFGP}_{x+1}}{\text{GP}_x} - \text{1 Year Term Premium}$$

Where 1 Year Term Premium is the first year benefit cost considering the timing of the benefit and premium in the first year.

For the high premium case,

$$\text{Expense Allowance} = \frac{(\text{DB} \times P_{x+1}) - \text{1 Year Term Premium}}{\text{PVFGP}_x}$$

Where 1 Year Term Premium is the first year benefit cost considering the timing of the benefit and premium in the first year.

If the timing is continuous or discounted continuous, then the expense allowance is multiplied by $$\frac{D_x}{\bar{D}_x}$$.

1.2.3 Illinois Method

The Illinois Method is the same as the Unitary CRVM method except the denominator in the expense allowance calculation is limited to the first 20 years (PVFGP$_x$). Thus, rather than calculating the present value of the gross premiums for the entire premium paying period, the present value calculation is limited to 20 years if the premium paying period is greater than 20 years. If the premium paying period is less than or equal to 20 years, then the Illinois method will produce results identical to the Unitary CRVM method.

1.2.3.4 Grading and the New Jersey Method

The New Jersey method is a special case of the grading method where the grading period is set to be 20 years. The net premium will grade from duration 2 using the unitary CRVM method to the end of the grading period. The net premium is set to the net level premium after the grading period. Generally, the formula for the graded modified net premium is

$$\beta = P + \frac{P - c}{a_{x+y+1}}$$

Where y+1 is the ending duration of the grading period. y = 19 for the New Jersey Method.

1.2.3 First year Modified net premium calculation ($\alpha$)

$\alpha$ is calculated such that the terminal reserve will be zero at duration zero. For the net level premium method, $\alpha = \beta$. For the modified premium methods, $\alpha$ is always constrained to be $\leq \beta$. If the first year terminal reserve is 0, then $\alpha$ will be the cost for a One Year Term insurance. The first year terminal generally will be zero if the policy is classified as a low premium policy (Full Preliminary Term is allowed) AND the expense allowance timing assumption is consistent with the policy timing assumption. If either of these conditions are not met, then the terminal reserve in the first year will be greater than 0 and that extra reserve will get added to the first year premium $\alpha$.

$$\alpha = (\text{Reserves}_{x+1} \times D_{x+1} + \text{Death Benefit}_{x+1} \times C_x^B + \text{Endowment}_{x+1} \times D_{x+1}) / D_x^A$$

$^A$ $D_x$ becomes $\bar{D}_x$ if continuous premium payments.

$^B$ $C_x$ becomes $C_x$ if immediate payment of claims (continuous or semi-continuous assumptions).

1.3 Calculate Terminal Reserves

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The terminal reserve calculation uses the recursive formula starting with the benefit cease age and ending with the issue age. This function will be called immediately after the net premium function is called. Similar to the net premium function, the terminal reserve function will be called for each segment and reserve basis (e.g. basic vs. minimum and unitary vs. segmented). The general formula is as follows:

\[ V_{k-1} = b_k \nu q_{x+k-1} + v_k \nu p_{x+k-1} - NP_{k-1} \]

Where \( NP_{k-1} \) is the net premium for duration \( k-1 \)

The benefit and net premium terms are adjusted if the timing assumption is not curtate.

The benefit adjustment is \( \frac{i}{\delta} \)

The premium adjustment is \( \frac{D_x}{D_x} \)

For the XXX YRT/ART method, \( kV \) is always 0.

### 1.4 Deficiency Reserves Calculation

Sources: Section 4C and 5B of the XXX Regulation

The interest rate and mortality still should be that was used for the minimum calculations unless assuming the YRT method which will use basic

Deficiency reserves are calculated for basic ART, Minimum Unitary, and Minimum Segmented. Deficiency reserves follow the general recursive reserve formula starting at maturity and calculating to the issue age. The general formula is:

\[ V_{k-1} = b_k \nu q_{x+k-1} + v_k \nu p_{x+k-1} - NP_{k-1} \]

Note that for deficiency reserves, the “benefit”, \( b \), is \( \text{Max}(0, \text{Net Premium} – \text{Gross Premium}) \). Also, in the recursive formula, there is no “net premium” so the last term is always 0. This simplifies to:

\[ V_k = \text{Max}(0, NP_k - GP_k) \nu q_{x+k-1} + v_k \nu p_{x+k-1} \]

An adjustment, \( \frac{D_x}{D_x} \), is made to the premium deficiency if the premium timing is continuous.

If there is no premium deficiency for the minimum unitary, then the minimum unitary reserve is set equal to the basic unitary. Likewise, if there is no premium deficiency for the minimum segmented, then the minimum unitary reserve is set equal to the basic segmented.

### Section 2: Gross Premiums

A recursive formula is used to calculate the present value of future gross premiums and the annuity factor. This values are calculated starting with premium cease age – 1 and ending with the issue age.

\[ \text{PVFGP}_{(x)} = \text{Gross Premium}_{(x \text{ - Issue Age})} \times \text{Premium Timing} + [(1 - \text{Final Mortality}_{x}) \times \text{PVFGP}_{(x+1)}] / (1 + i) \]

\[ \tilde{a}_x = 1 \times \text{Premium Timing} + [(1 - \text{Final Mortality}_{x}) \times \tilde{a}_{x+1}] / (1 + i) \]

The Premium Timing adjustment, \( \frac{D_x}{D_x} \), is made to the formulae above if the premium timing is continuous.

The policy fee is included in the gross premium for the basic and/or minimum calculations based upon the user input choice.

All calculations are on a per unit basis.
Section 3: Commutation Functions

The commutations functions are recalculated every time the mortality changes. For XXX policies, this will be for the ½ Cx, Basic, and Minimum assumptions. Segmentation does NOT change mortality compared to unitary calculations.

\[ l_{\text{Issue Age}} = D_{\text{Issue Age}} = 1000000 \]
\[ \text{Discount } l_{\text{Issue Age}} = 1 \]

Calculate the following for all ages (from issue age to benefit cease age)

- \( i \) equals either the basic or the minimum interest rate
- Final Mortality incorporates all of the select and X Factors

\[
\begin{align*}
\text{Discount}_{x+1} &= \text{Discount}_x / (1 + i) \\
l_{(x+1)} &= l_x * (1 - \text{Final Mortality}_x) \\
D_{(x+1)} &= l_{(x+1)} * \text{Discount} \\
C_x &= D_x * \text{Final Mortality}_x / (1 + i) \\
\bar{C}_x &= C_x * i / \delta_i \\
\end{align*}
\]

where \( \delta_i = \ln(1+i) \)

Calculate the following for all ages (from benefit cease age to issue age)

\[
\begin{align*}
\bar{D}_x &= 0.5 * \left( D_{(x+1)} + D_x \right) \\
M_x &= M_{(x+1)} + C_x \\
\bar{M}_x &= \bar{M}_{(x+1)} + \bar{C}_x \\
N_x &= N_{(x+1)} + D_x \\
\bar{N}_x &= \bar{N}_{(x+1)} + \bar{D}_x \\
\end{align*}
\]

Section 4: Benefits

The present value of future benefits is calculated as a recursive formula starting at the ending period and ending at the beginning period. It is recalculated each time the mortality changes (1/2 Cx, Basic, or Minimum) AND it is recalculated for unitary vs. segmentation. The end of the period is the benefit cease age for a unitary calculation and the end of the segment period for a segmentation option. The present value of future benefits at any duration is the sum of the present value of future death benefits and the present value of future pure endowments.

\[ PVFB_{x+k-1} = PVFDB_{x+k-1} + PVFPE_{x+k-1} \]

4.1 Death Benefit

The death benefit at any duration is based upon the user input. If the user selects the “ROP – DB & CV” return of premium option, then the percentage of accumulated premiums are added to the death benefit. The death benefit, \( DB_k \), at any duration \( k \) is:

\[ DB_k = \text{Death Benefit}_k + \text{ROP\%}_k * \sum_{j=1}^{k} \left( \text{Gross Premium}_j + \text{Indicator}_j * \frac{\text{Policy Fee}}{\text{Number of Units}} \right) \]

Where Death Benefit\(_k\) is the death benefit the user entered at duration \( k \), ROP\% is the return of premium percentage (entered as a decimal) that the user entered at duration \( k \)
Gross Premium, $j$ is the gross premium per unit the user entered for duration $j$. Unless the system is processing a current premium Actuarial Guideline CCC test, the current premium is the guaranteed premium. Indicator is 1 if the user chooses to include the policy fee in the premium returned and 0 otherwise. Policy Fee and Number of Units are the values entered by the user.

If the user did not choose a return of premium option or the return of premium is not included in the death benefit, then the previous formula simplifies to:

$$DB_k = \text{Death Benefit}_k$$

The following is calculated for issue age $x$ and all durations $k$.

$$PVFDB_{x+k-1} = DB_k v_{x+k-1} \text{TimingAdj}_k + PVFDB_{x+k} v_p$$

Where the TimingAdj$_k$ is $i/\delta$ if the timing is not curtate. The starting value for PVFDB at the end of the period is 0.

### 4.2 Endowment Amount

If the user chooses a return of premium option, then for the duration, $k$, of the endowment period, the endowment amount ($\text{Endow}_k$) is:

$$\text{Endow}_k = \text{Max}\{\text{Endowment}_k, \text{ROP}\%_k * \sum_{j=1}^{k} (\text{Gross Premium}_j + \text{Indicator} * \frac{\text{Policy Fee}}{\text{Number of Units}})\}$$

Where Endowment$_k$ is the Endowment the user entered at duration $k$, ROP% is the return of premium percentage (entered as a decimal) that the user entered at duration $k$, Gross Premium$_j$ is the gross premium per unit the user entered for duration $j$. Unless the system is processing a current premium Actuarial Guideline CCC test, the current premium is the guaranteed premium. Indicator is 1 if the user chooses to include the policy fee in the premium returned and 0 otherwise. Policy Fee and Number of Units are the values entered by the user.

For all other conditions and durations, the endowment amount ($\text{Endow}_k$) is:

$$\text{Endow}_k = \text{Endowment}_k$$

If the user chooses to endow the cash value or the reserve (minimum or basic), then the endowment amount at the end of the period will be the maximum of the endowment amount the user enters and the cash value or reserve option the user chooses. If it is not at the end of period duration, the endowment amount is the amount the user entered if any. The following present value of future pure endowments is calculated for issue age $x$ and all durations $k$.

$$PVFPE_{x+k-1} = \text{Endow}_{k-1} + PVFPE_{x+k} v_p$$

The starting value for PVFPE at the end of the period is the pure endowment amount at the end of the period, if any. There is no timing adjustment because all pure endowments are assumed to occur at the end of the year.

### Section 5: Cash Values

The cash value calculation uses the recursive formula starting with the benefit cease age and ending with the issue age. The general formula for the cash value is similar to the general formula for the terminal reserve except the adjusted premium is substituted for the net premium. The general formula the terminal cash values at duration $k-1$ is as follows:

$$k-1 CV_x = b_k v_{x+k-1} + k CV_x v_{x+k-1} - P_{k-1}^{Adj}$$
Where \( P^{adj}_{k-1} \) is the adjusted premium for duration \( k-1 \).

The benefit and adjusted premium terms are adjusted if the timing assumption is not curtate.

The benefit adjustment is \( \frac{D}{\delta} \)

The premium adjustment is \( \frac{D}{\delta} \)

If the user chooses the Actuarial Guideline CCC method, then the final minimum cash value will be the largest minimum cash value by duration based upon the following tests:

1. Pure 1980 SNFL assumptions. The benefit period is the based upon the benefit cease age from the user input (Unitary Test). The premiums assumed are the guaranteed premiums.
2. AG CCC assumptions. The benefit period is the endowment period as defined by the user. The primary benefit is not used. The only death benefit used is the return of premiums if the user selects the “ROP – DB & CV” return of premium option. Otherwise, there are no death benefits. The Average Amount of Insurance used in the expense allowance calculation is the same AAI used in the expense allowance for Test 1: Pure 1980 SNFL assumptions. The premiums assumed are the guaranteed premiums.
3. Same as Test 1 except the current premiums are used rather than the guaranteed premiums. This test is only performed if the user selects the “Test Current Premiums” option.
4. Same as Test 2 except the current premiums are used rather than the guaranteed premiums. This test is only performed if the user selects the “Test Current Premiums” option.

This adjusted premium depends upon which method the user selects.

5.1 Adjusted premium using the 1980 or the Actuarial Guideline CCC Method

If the Cash Value Method is the 1980 Method, then

\[
\text{Avg Amount of Insurance (AAI)} = \frac{1}{10} \sum_{x=1}^{10} DB_{\text{BeginAge} + x - 1}
\]

Note that the AAI for the endowment test for AG CCC is the same AAI used for the Unitary Test.

A Net premium factor is calculate as:

\[
\text{Net Premium} = \frac{\text{PVFB}_{\text{Begin Age}}}{(\text{N}_{\text{Begin Age}} - \text{N}_{\text{Premium Cease Age}}) / \text{D}_{\text{Begin Age}}}
\]

The adjusted premium calculation for the AG CCC method is similar to the 1980 SNFL process. The main difference for each “test” for the AG CCC method, the benefits and benefit cease period varies.

See the Death Benefit and Endowment Amount section for details on the definitions for the benefits used in the PVFB (Present Value of Future Benefit) factor and other benefit calculations.

If Expense Allowance Curtate is NOT selected and the timing is continuous or discounted continuous, then \( N_x \) is replaced by \( \overline{N}_x \) in the previous formula.

The cash value “expense allowance” is calculated as:

If Net Premium < 0.04 * Avg Amount of Insurance, Then

\[
\text{CVEA} = 1.25 \times \text{Net Premium} + 0.01 \times \text{Avg Amount of Insurance (AAI)}
\]

Else

\[
\text{CVEA} = 0.06 \times \text{Avg Amount of Insurance (AAI)}
\]

The adjusted premium is a constant percentage, \( r_c \), of the gross premiums where

\[
r_c = \frac{\text{PVFB}_{\text{Begin Age}} + \text{CVEA}}{\text{PVFGP}_{\text{Begin Age}}}
\]
5.2 Adjusted premium using the 1941 Method

If the Cash Value Method is the 1941 Method, then a circular formula results. A constant premium and constant death benefit is assumed. An adjusted premium “test” is performed to determine the expense allowance allowed.

The adjusted premium test is as follows:

\[
\begin{align*}
\text{PVFB Test} &= \frac{M_{\text{Begin Age}}}{D_{\text{Begin Age}}} \\
\text{Annuity} &= \frac{N_{\text{Begin Age}}}{D_{\text{Begin Age}}} \\
\text{Adjust Premium Test}_0 &= \frac{(\text{PVFB Test} + 0.02)}{(\text{Annuity} - 0.65)} \\
\text{If Adjust Premium Test}_0 &> 0.04 \text{ Then recalculate} \\
\text{Adjust Premium Test}_0 &= \frac{(\text{PVFB Test} + 0.046)}{\text{Annuity}} \\
\text{PVFB Per Unit} &= \frac{\text{PVFB}_{\text{Begin Age}}}{\text{Death Benefit}} \text{ where PVFB}_{\text{Begin Age}} \text{ includes any endowments.} \\
\text{Annuity} &= \frac{(N_{\text{Begin Age}} - N_{\text{Premium Cease Age}})}{D_{\text{Begin Age}}} \\
\text{If the timing is continuous or discounted continuous, then } N_x \text{ is replaced by } \frac{N_x}{2} \text{ in the previous formula.}
\end{align*}
\]

If Adjust Premium Test\(_0\) < 0.04, Then

\[
\text{Adjust Premium Test}_1 = \frac{(\text{PVFB Per Unit} + 0.02)}{(\text{Annuity} - 0.65)} \\
\text{If Adjust Premium Test}_1 > \text{Adjust Premium Test}_0, \text{ Then} \\
\text{Adjust Premium Test}_1 = \frac{(\text{PVFB Per Unit} + 0.02 + 0.25 \times \text{Adjust Premium Test}_0)}{(\text{Annuity} - 0.4)} \\
\text{If Adjust Premium Test}_1 \geq 0.04, \text{ Then} \\
\text{Adjust Premium Test}_1 = \frac{(\text{PVFB Per Unit} + 0.036 + 0.25 \times \text{Adjust Premium Test}_0)}{(\text{Annuity})}
\]

Else

\[
\begin{align*}
\text{Adjust Premium Test}_1 &= \frac{(\text{PVFB Per Unit} + 0.02)}{(\text{Annuity} - 0.65)} \\
\text{If Adjust Premium Test}_1 &\geq 0.04, \text{ Then} \\
\text{Adjust Premium Test}_1 &= \frac{(\text{PVFB Per Unit} + 0.046)}{\text{Annuity}}
\end{align*}
\]

Final Adjusted Premium\(_x\) = Adjusted Premium\(_1\) \times \text{Death Benefit Constant}

Note that Final Adjusted Premium\(_x\) is 0 for \(x \geq\) the premium cease age.

5.3 Modify for Minimum and percentage

If the cash values that the user inputs are less than the minimum cash values, then the final cash values are increased to the minimum cash values if the user selects this option.

Also, the user has the option to make the cash values to be a percentage of the accumulated gross premiums. For example, if the cash values are 10.34% of the accumulated premiums, Actuarial Auditor will round the cash values up so that they would be 11% of the accumulated gross premiums. This modification of the cash values is only performed if the user selects this option.

Section 6: Mid Year and Comparisons

After the net premiums and terminal reserves are calculated for each of the mortality, interest rate and segmentation basis, mid year reserves are calculated. For the XXX method, comparisons are made to determine the final values (net premiums, terminal and mid year reserves) are used.

6.1 Interpolation

The following interpolation factors are used in this section

\[
\begin{align*}
\text{If Mean Interpolation} &= \text{Half Year, Then} \\
\text{Interpolation Factor} &= 0.5 \\
\text{If Mean Interpolation} &= \text{Next Month, Then} \\
\text{Interpolation Factor} &= 1 - \left(\frac{\text{Month Duration}}{12}\right) \\
\text{If Mean Interpolation} &= \text{Exact Day, Then} \\
\text{Interpolation Factor} &= \text{exact number of days from valuation date to issue date} / \text{total number of days in the year}
\end{align*}
\]
6.2 MidYear Reserve

The general formula for the mid year reserve, $kMV_x$ for issue age $x$, duration $k$ is:

$$kMV_x = t \left( \frac{1}{k - 1} V_x + NP_{x+k-1} \right) + (1-t) \frac{1}{k} V_x$$

Where $t$ is the interpolation factor defined above.

If the mid year reserve definition is mid terminals, then the term $NP_{x+k-1}$ does not exist in the formula.

For cash values, the terminal reserves are replaced by the terminal cash values and the net premium is replaced by the adjusted premium.

If the rounding order is to round the terminals and net premium first, then the terminals reserves and net (adjusted) premiums are rounded before the mid year formula is applied.

If the timing is continuous discounted mean, then the net premium is multiplied by $d/\delta$. The net premium that is printed on the summary page is not affected by this adjustment.

6.3 XXX Comparisons

Terminal and mid year Reserves and net premiums are calculated for every mortality assumption (1/2 Cx, basic, and minimum) and for unitary and segmentation (basic & minimum). Given all these assumptions, comparisons need to be made to determine the final premiums and reserves. These comparisons are as follows:

6.3.1 Mid Year, Basic Reserve

This reserve is the maximum of the $1/2$ Cx reserve (if selected) and the net premium from either the unitary or segmentation. The user selects whether the comparison should be made between the terminals or the mid year reserves. The unitary or segmented net premium is selected based upon which reserve is greater. Also, if the user selects to test for unusual cash values, then the premium from that calculation is also part of the comparison of the greatest net premium.

6.3.2 Mid Year, Minimum Reserve

The calculation for the mid year minimum reserve is similar to the mid year basic reserve except that the $1/2$ Cx and test for unusual cash values are not included.

6.3.3 Terminals, Basic and Minimum Reserve

The user selects whether the comparison is made between the mid year or the terminals to determine whether the unitary or segmented reserves are used. Only applicable for XXX method. This minimum comparison ignores deficiency reserves.

6.3.4 Net Premium, Basic and Minimum assumptions

The criteria to choose which net premium to display follows similar logic as the criteria to choose the mid year reserves. For XXX methods, the user chooses whether to compare mid year or terminal reserves. Based upon that selection, the appropriate reserves are compared. The basic and minimum net premiums, either unitary or segmented, is selected based upon whether the unitary or segmented reserve is larger. This minimum comparison ignores deficiency reserves.

6.3.5 Deficiency Reserves

The terminal deficiency reserve is defined as the maximum of 0 and the difference between the minimum terminal (without deficiencies) and the basic terminal. Once the terminal deficiency reserve is calculated, it is added to the minimum terminal reserve.

Section 7: Segmentation (Reserves)

For the unitary methods, there is only one segment from the issue age to the benefit cease age. For the segmentation methods in XXX, Segments are calculated once and are used for both the basic and minimum mortality assumptions. Basic mortality assumptions are used to determine the segments.

7.1 Gross Premium ratios

The user chooses whether to include the policy fee in the gross premium ratio calculation. Calculations are performed on a per unit basis. For all $x$ from the issue age to the benefit cease age – 2, the following are calculated:

$$\text{Gross Premium Ratios}_x = \frac{(\text{Premium}_{x+1} \div \text{Death Benefit}_{x+1})}{(\text{Premium}_x \div \text{Death Benefit}_x)}$$

$$\text{Gross Premium Segment}_x = \text{Premium}_0$$
If \( x = \text{Benefit Cease Age} - 2 \), Then
* Gross Premium Segment\( (x + 1) = \text{Premium}_1 \)
* Gross Premium Ratios\( (x + 1) = 1 \)
* Mortality Ratios\( (x + 1) = 1 \)
End If

### 7.2 Mortality Ratios

Mortality Ratios\( x = \frac{q_{x+1}}{q_x} \)

The mortality ratio can be modified based upon user input,
* If Segment Definition = Add 1 Pct, Then
  * Mortality Ratios\( x = \text{Mortality Ratios}_x + 0.01 \)
* Else If Segment Definition = Multiply By 1.01, Then
  * Mortality Ratios\( x = \text{Mortality Ratios}_x * 1.01 \)
* Else If Segment Definition = Subtract 1 Pct, Then
  * Mortality Ratios\( x = \text{Mortality Ratios}_x - 0.01 \)
* Else If Segment Definition = Multiply By 99, Then
  * Mortality Ratios\( x = \text{Mortality Ratios}_x * 0.99 \)
End If

Mortality Ratios\( x \) are never greater than one.

### 7.3 Segmentation test

For all \( x \) from the benefit cease age -1 to the issue age, the following test is performed

A new segment is created when Gross Premium Ratios\( x > \text{Mortality Ratios}_x \).

### Section 8: Mortality

#### 8.1 General formula

For XXX methods, the mortality rates are calculated for basic, \( \frac{1}{2} Cx \), and minimum assumptions. The same mortality is used for unitary and segmentation. For each life, the following process is followed to calculate the final mortality:

The following are calculated for each \( x \) starting at the issue age to the benefit cease age:

For \( \frac{1}{2} Cx \), if the duration is \( \leq 10 \), the select factor is used, if any.
For basic and minimum assumptions, the select factor is used if it is still in the first segment. The X Factor is also added for the minimum assumption in the first segment, if applicable. If it is a renewal segment, then the user defines whether the select factor is used for the basic and/or minimum assumptions. The select factors will never be used past 10 years in the renewal segments.

Final \( q_x = q_x * \text{Select Factor} \), where the Select Factor will equal 1 if not meeting any of the above criteria.

#### 8.2 Last Survivor two lives

If there are two lives, a last survivor assumption is made and a final \( q \) is calculated using the Fraserized Method. The method is as follows:

For all ages \( x \), starting from the lowest issue age and ending at the benefit cease age, let \( 0p_x^1 = 0p_x^2 = 1 \) when \( x = \) the lowest issue age.
For all durations, \( k \), define

\[
\begin{align*}
\text{kP}_x^1 &= \text{kP}_x^1 (1 - q_x^1) \\
\text{kP}_x^2 &= \text{kP}_x^2 (1 - q_x^2)
\end{align*}
\]

The probability that both survive is defined as \( \text{kP}_x^B = \text{kP}_x^1 \text{kP}_x^2 \)

The probability that only life 1 survives is defined as \( \text{kP}_x^{1\text{only}} = \text{kP}_x^1 (1 - \text{kP}_x^2) \)

The probability that only life 2 survives is defined as \( \text{kP}_x^{2\text{only}} = \text{kP}_x^2 (1 - \text{kP}_x^1) \)
The probability that either life survives is defined as 

\[ k_p^E = k_p^B + k_p^{1\text{Only}} + k_p^{2\text{Only}} \]

The last survivor mortality is defined as 

\[ q_{x+k}^{LS} = 1 - \left( \frac{k_p^E}{k_p^{x-1}} \right) \]

Note: Logic exists to implement a first to die method but is not implemented due to lack of interest.

Section 9: Unusual Cash Value Test

In order to ensure adequate funding for significant increases in guaranteed cash values, an unusual pattern of cash value floor is calculated according to section 6D of the XXX regulation. Basic mortality assumptions are used throughout this test. A policy is considered to have unusual cash values if the change in the cash values is greater than \(1.1 \times GP_t + 1.1 \times (CSV_{t+1} + GP_t) \times iNF + 0.05 \times SC_1\). For every occurrence of an unusual cash value, a new segment is created.

9.1 Expense Allowance

An expense allowance is calculated as follows:

\[ NP_{Adj} = \frac{\text{Final Gross Premiums}_0 \times \text{Unusual CV PV Segment Benefits}_{(x+1)} \times \text{Unusual CV PV Segment Premium}_{(x+1)}}{19 \text{ Pay Premium} = \text{Renewal} 19 \text{ Pay Premium} \times \text{Equivalent Level Renewal Account}} \]

\[ 1^\text{st} \text{ Year Benefit Premium} = \text{Final Mortality}_{(x-1)} \times \text{Death Benefit}_{(x-1)} \times \text{Death Benefit Timing} / (1 + \text{Final Interest Rate}_0) \]

The 1\text{st} year benefit premium adjustment is \(\frac{\hat{D}_x}{\check{D}_x}\) for continuous premiums.

\[ \text{Exp Allow} = (\text{Min(Adjusted Net Level Premium, 19 Pay Premium)} - 1^\text{st} \text{ Year Benefit Premium}) \]

The Exp Allow is adjusted by \(\frac{\hat{D}_x}{\check{D}_x}\) for continuous premiums.

For each \(x\) from the benefit cease age to the issue age + 1, the following are calculated:

\[ \text{Unusual Cash Value PV Segment Premiums}_x = (D_{(x-1)} \times PVFGP(x-1) - D_y) \times PVFGP_y) / D_{(x-1)} \]

\[ \text{Unusual Cash Value PV Segment Benefits}_x = (D_{(x-1)} \times PVFB \text{ No Endowment}_{(x-1)} - D_y \times PVFB \text{ No Endowment}_y / D_{(x-1)} + \text{Expense Allowance} + \text{Cash Value}_y \times D_y / D_{(x-1)} \]

Where \(y\) is the attained age of the current segment.

Unusual Cash Value Terminal Reserve\_Issue\_Age = -Exp Allow

9.2 Terminal Reserve

Once each segment is determined, the present value of future premiums and segments are calculated for each segment using basic principles. A net premium is calculated from these present values that is defined as a constant percentage of the gross premium for each segment. With the net premium, a traditional terminal reserve using the recursive method is calculated:

\[ k-1 \times V_x = b_k x q_{x+k-1} + b_k x v p_{x+k-1} - NP_{k-1} \]

Standard benefit timing adjustment is made for immediate payment of claims and premium adjustment if the premium timing is continuous.

9.3 Mid Year Reserve

If the mid year assumption is the mean reserves then an interpolated value of the net premium is added to an interpolated sum of the terminal reserves using basic principles to calculate the mid year reserve. Mid year rounding rules determine the rounding definition.

9.4 Unusual Cash Value Test

The maximum of the mid year reserve and the cash value is the final value that gets compared to the basic mean reserve to determine the greatest basic mean reserve.
Section 10: Smoothness Test

10.1 Nonforfeiture Factors (NFF)
The NFF used in the smoothness test are constant percentages of the Adjusted Premiums (See Adjusted Premium 1980 Method) used in the minimum cash value test. Because each test in the AG CCC method uses a different set of adjusted premiums, then a decision must be made to determine which set of adjusted premiums to use for the smoothness test. Based upon the authors of the AG CCC, the intent was that the adjusted premiums that would be used for the smoothness test would be the ones used in the 1980 SNFL minimum test.

The NFF are defined for each segment as the constant percentage of the adjusted premiums such that the basic cash value will equal the cash value from the return of premium for that duration.

10.2 Basic Cash Value Calculation
The basic cash value calculation uses the same process as the minimum cash value calculations using NFF rather than Adjusted Premiums. However, the calculation is not performed on a unitary basis but on a segmented basis where the segments are defined in the following section.

10.3 Segments
The user selects the duration of the first segment. If the user chooses the 5 year option, then the first segment will be 5 years. If the user chooses the Variable option, then the system will start with a 5 year first duration and try to find a set of NFF that meet the smoothness test. If 5 years doesn’t work, then the first segment is set to 6 years. The process continues until a set of NFF are found that pass the smoothness test or the maximum first segment duration is reached. The maximum first segment duration is defined as:

Maximum First Segment Duration = Max(5, the first duration where the actual cash value reaches $2 per thousand of insurance)

Every duration after the first duration is 5 years with the possible exception of the last duration. If the last duration until the endowment period is not exactly 5 years, then it is defined to be between 6 and 9 years so that the last segment ends at the endowment period.

10.4 Smoothness Test Determination
For each duration in the endowment period, a calculation is made to determine the absolute difference between the basic cash value and the cash value resulting from the return of premiums (actual cash value). A comparison is made at each duration to identify whether this absolute difference is less than the threshold. If it is less than the threshold, then the smoothness test passes for that duration. If the smoothness test fails at ANY duration, then the smoothness test overall fails. The threshold used is 0.002 * AAI (Average Amount of Insurance).

10.5 Modify for Smoothness
Actuarial Auditor uses an optimized algorithm to determine how to efficiently modify the cash values so that they meet smoothness requirements. The goal is to minimize the amount that the cash values must be increased in order to pass the smoothness test.

Section 11: Selected References
The following sources were reviewed. Generally, Actuarial Auditor is consistent with these sources. However, except for the “Actuarial Guidelines” and “Valuation of Life Insurance Policies Model Regulation”, intended differences may exist due to methodology or interpretive variances.

“Actuarial Guidelines”, National Association of Insurance Commissioners Actuarial Task Force


“Life Contingencies”, by C. W. Jordan


“Valuation of Life Insurance Policies Model Regulation”, National Association of Insurance Commissioners


Index
19 Pay Death Benefit Definition, 4
19 Pay Renewal Death Premium, 4
Adjusted Premium, 10
Adjusted Premium using the 1941 Method, 11
Adjusted Premium using the 1980 Method, 10
Adjusted Premium assuming Actuarial Guideline CCC, 10
Alpha, 6
Average Death Benefit, 4
Average Amount of Insurance, 10
Benefits, 8
Commutation Functions, 8
Continuous Discounted Mean, 12
Cash Values, 9
Death Benefit, 8
Death Benefit Used in Expense Allowance, 4, 5
Deficiency Reserves, 7, 12
ELRA, 4
Endowment Amount, 9
Expense Allowance, 3, 6
Expense Allowance Timing, 5
Final Mortality, 8
Frazerized Method, 13
Full Preliminary Term, 5
Gross Premium Ratios, 12
Grading Method, 6
High Premium, 4, 5, 6
Illinois Method, 6
Interpolation, 11
Last Survivor Two Lives, 13
Low Premium, 4, 5, 6
Menge Interpretation, 5
Mid Year Reserve, 12
Mortality Ratios, 13
New Jersey Method, 6
Policy Fee, 7, 12
Segmentation Assumptions, 3, 12
Segmentation Test, 13
Select Factors, 8, 13
Smoothness Test, 15
Terminal Reserves, 6
Unitary Assumptions, 3
Unitary CRVM, 6
Unusual Cash Values Test, 14
X Factors, 8, 13
XXX Comparisons, 11, 12